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Quantum theory of non-ideal photon detectors

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Abstract. We study the signal emitted by a narrow-band photon detector which receives a broad-band electromagnetic field. The statistical properties of the signal are shown to be completely different from those in the usual case of an ideal photon detector receiving a quasimonochromatic field.

1. Introduction

It is now proved that the photoelectrons emitted by a usual detector are set as a compound Poisson distribution.

This result was a postulate in classical theory (Mandel and Wolf 1965) but has been proved with the help of quantum description of the detection mechanism by Glauber (1965) in the case of independent atoms. A full quantum theory taking account of interactions between atoms inside the detector was recently presented by Rocca (1971), who showed that the corrective terms are negligible in practical counting experiments (Arnedo and Rocca 1974). Therefore we shall consider independent atoms and use Glauber's description. Ideal photodetectors act through a one-photon process, by atomic ionization (or excitation in semiconductors), without any intermediate stage, and the spectral width $\Delta\omega$ of the detector sensitivity function $S(\omega)$ (Glauber 1965) is much broader than the spectral width $\delta\omega$ of the incident light (cf figure 1(*a*)). In that case Glauber showed that the temporal density of the photoelectron Poisson process is, for a quasimonochromatic incident field

$$I(t) \propto |\mathscr{E}(t)|^2 \tag{1}$$

where $\mathscr{E}(t)$ is the analytic signal (Born and Wolf 1964) associated with the incident field, ie the eigenfunction of the positive frequency part of the EM field. In this paper we still suppose independent atoms but we do not restrict ourselves to the case of ideal photodetector for which $\delta\omega \ll \Delta\omega$. For arbitrary value of $\delta\omega$ and $\Delta\omega$, we prove that the photoelectrons are still distributed as a compound Poisson process and we derive its density. We thus prove that we do not obtain the same results if we detect a broad-band signal with:

(*a*) an ideal broad-band photodetector, after filtering the signal (this is Glauber case), see figures 1(*a*) and 2(*a*), $\delta\omega \ll \Delta\omega$; as we would with:

(*b*) a narrow-band detector, without any filtering before the detection process, see figures 1(*b*) and 2(*b*), $\delta\omega \gg \Delta\omega$.

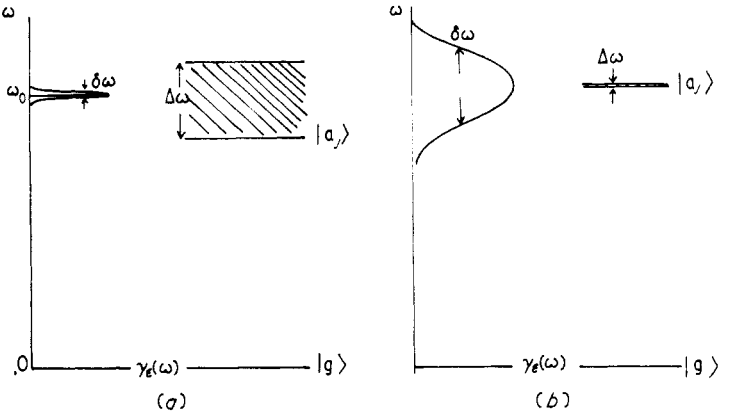

Figure 1. Spectral distribution $\gamma_{\mathscr{E}}(\omega)$ of the EM field with regard to the atomic ionization levels $|a_j\rangle$ of the detector. (*a*) A broad-band detector receives a narrow-band signal, $\Delta\omega \gg \delta\omega$. (*b*) A narrow-band detector receives a broad-band signal, $\Delta\omega \ll \delta\omega$.

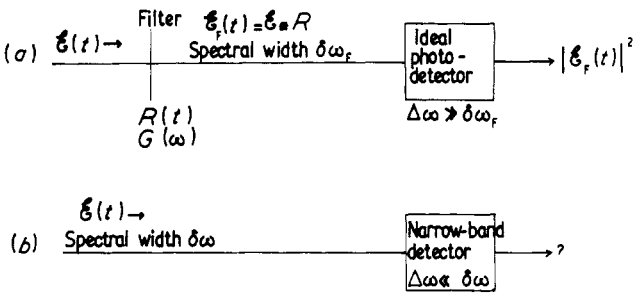

Figure 2. (*a*) An ideal photodetector, with a broad-band ionization level, receives a quasi-monochromatic field. The incident field $\mathscr{E}(t)$ is filtered before the detector, so that $\mathscr{E}_F(t)$ has a spectral width $\delta\omega_F \ll \Delta\omega$. The signal is $i(t) = |\mathscr{E}_F(t)|^2$. (*b*) A non-ideal photodetector receives a broad-band EM field. How is the signal?

2. Ionization rate of a non-ideal detector receiving an arbitrary EM field

Let us apply the quantum theory of atomic detectors when we suppress the two restricting hypotheses introduced by Glauber (1965):

$$\begin{aligned} &\delta\omega \gg \omega_0, && \text{monochromatic field } (G_1) \\ &\Delta\omega \gg \delta\omega, && \text{ideal photodetector } (G_2). \end{aligned} \tag{2}$$

Hereafter we will consider two arbitrary spectral widths $\delta\omega$ and $\Delta\omega$. We assume that photoelectrons are emitted

$$\text{only by one-photon processes, } (G_3) \tag{3}$$

ie there is no resonant intermediate level which is able to participate in the ionization process, and the incident flux is not sufficient to introduce multiphoton processes. Let us

apply perturbation theory to the system detector plus EM field. The interaction Hamiltonian between the atom and the field is

$$H_i(t) = \exp\left(i\frac{\mathscr{H}_0}{\hbar}t\right)H_i \exp\left(-i\frac{\mathscr{H}_0}{\hbar}t\right) = -e\sum_\gamma \boldsymbol{q}_\gamma(t)\cdot\boldsymbol{E}(\boldsymbol{r}, t). \tag{4}$$

According to the first-order perturbation theory, the state of the system at time t can be deduced from the state at time t_0 using the relation

$$|t\rangle = U(t, t_0)|t_0\rangle = \left(1+\frac{1}{i\hbar}\int_{t_0}^{t}\mathscr{H}_i(t')\,dt'\right)|t_0\rangle. \tag{5}$$

The transition probability amplitude from the initial state $|gi\rangle$ of the system at time t_0 to the final state $|af\rangle$ at time t, where g and a refer to atoms, i and f to the field, is

$$\langle af|U(t, t_0)|gi\rangle = \frac{ie}{\hbar}\sum_\gamma\int_{t_0}^{t}\langle a|q_\gamma(t')|g\rangle\langle f|E(r, t')|i\rangle\,dt', \tag{6}$$

where

$$q_\gamma(t') = \exp\left(i\mathscr{H}'_{0,at}\frac{t'}{\hbar}\right)q_\gamma(0)\exp\left(-i\mathscr{H}_{0,at}\frac{t'}{\hbar}\right). \tag{7}$$

$H_{0,at}$ represents the atomic part of the Hamiltonian at time t_0. If we introduce the atomic transition momentum

$$\langle a|\,\Sigma_\gamma\, q_\gamma(0)|g\rangle = M_{ag}$$

we have

$$\langle a|\,\Sigma_\gamma\, q_\gamma(t')|g\rangle = M_{ag}\exp(i\omega_{ag}t').$$

Equation (6) becomes

$$\langle af|U(t, t_0)|gi\rangle = \frac{ie}{\hbar}\int_{t_0}^{t}\exp(i\omega_{ag}t')M_{ag}\langle f|E(r, t')|i\rangle\,dt'. \tag{8}$$

In the right-hand side of equation (8), when we are interested in times t such that $t-t_0 \gg 1/\omega_{ag}$, the only frequencies which contribute to the integration are the positive ones. Whatever the value of the spectral width of the incident EM field (if condition G_3 is fulfilled) the expression $\int_{-\infty}^{+\infty}\exp[i(\omega_{ag}+\omega_k)t']\,dt'$ always vanishes when $(\omega_k+\omega_{ag})$ is strictly positive. We can thus put $E^+(r, t')$ in place of $E(r, t)$ in equation (8). This property corresponds to the energy conservation principle; ie the atom is ionized by absorbing one photon. Processes where excitation of the atom takes place with emission of a photon are thus eliminated.

The ionization probability $P^{(1)}(t)$ of an atom which absorbs one photon is thus, at time t, the mean value over all initial states $|i\rangle$ of the field, of the square of the transition probability amplitude $\Sigma_f|\langle af|U(t, t_0)|gi\rangle|^2$ weighted by a function $R(a)$ which depends on geometrical characteristics of the detector

$$P^{(1)}(t) = \sum_a R(a)P_{g\to a}(t) \tag{9}$$

where

$$P_{g\to a}(t) = \left(\frac{e}{\hbar}\right)^2 \sum_f \iint_{t_0}^{t} \exp[i\omega_{ag}(t''-t')]M_{ag}M_{ag}^* \operatorname{Tr}(\rho E^-(t')E^+(t''))\,dt'\,dt''. \tag{10}$$

Let us introduce the spectral response function of the detector

$$S(\omega) = \frac{2\pi e^2}{\hbar^2} \sum_a R(a)M_{ag}M_{ag}^*\,\delta(\omega-\omega_{ag}), \tag{11}$$

and its Fourier transform, which we call the atomic impulse function,

$$S(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S(\omega)\,e^{i\omega t}\,d\omega. \tag{12}$$

The atomic transition frequencies ω_{ag} are positive so that the function $S(\omega)$ characterizing the ionization level is nonzero only for $\omega > 0$ and the atomic impulse $S(t)$ satisfies

$$S(t) = S^*(-t). \tag{13}$$

The probability that the atom is excited at time t, if it received the light since time t_0, can thus be written

$$P^{(1)}(t) = \iint_{t_0}^{t} S(t''-t')G^{(1)}(t', t'')\,dt'\,dt'' \tag{14}$$

where $G^{(1)}(t', t'')$ is the first moment of the EM field (Glauber 1965)

$$G^{(1)}(t', t'') = \operatorname{Tr}(\rho E^-(t')E^+(t'')). \tag{15}$$

The ionization rate, or ionization probability per unit time is the time derivative of $P^{(1)}(t)$

$$w^{(1)}(t) = \int_{t_0}^{t} (S(t-t')G^{(1)}(t, t') + S(t'-t)G^{(1)}(t', t))\,dt' \tag{16}$$

where t_0 corresponds to the beginning of irradiation of the atom.

If the EM field is stationary

$$w^{(1)}(t) = \int_{t_0}^{t} (S(\theta)\Gamma_{\mathscr{E}}(\theta) + S(-\theta)\Gamma_{\mathscr{E}}(-\theta))\,d\theta, \tag{17}$$

in this equation we used the classical notation

$$\Gamma_{\mathscr{E}}(\theta) = \langle \mathscr{E}(t)\mathscr{E}^*(t-\theta)\rangle = G^{(1)}(t, t-\theta), \tag{18}$$

which implicitly uses the correspondence between the analytic signal $\mathscr{E}(t)$ (which is a random function), and the eigenvalue $\mathscr{E}(\alpha_k, t)$ of the field operator $E^-(t)$ in the quantum theory. Both descriptions are indeed identical when we use the P-representation (Glauber 1963)

$$\Gamma_{\mathscr{E}}(\theta) = \int\cdots\int_{-\infty}^{+\infty} P\{\alpha_k\}\mathscr{E}(\alpha_k, t)\mathscr{E}^*(\alpha_k, t-\theta)\prod_k d^2\alpha_k. \tag{19}$$

Let us compare the one-photon counting rate $w^{(1)}(t)$ in the two cases of figures 2(*a*) and

2(*b*); we detect the same stationary light, with spectral width $\delta\omega$, in the two following manners:

(*a*) with an ideal photodetector ($S(t) = \delta(t)$) using a filter of gain $G(\omega)$ which reduces the spectral width of the incident light;

(*b*) with a narrow-band detector without any filter.

We shall take the atomic impulse such that

$$S(\omega) + S^*(\omega) = |G(\omega)|^2. \tag{20}$$

In these two experiments the initial field is filtered, but in the first case it is before detection, in the second case the detector acts as a filter.

The counting rates are given in case (*a*) by equation (4.22) of Glauber (1965), and in case (*b*) by equations (17), (18) of this paper

$$w_a^{(1)}(t) = G^{(1)}(t, t) = \int_{-\infty}^{+\infty} \gamma_{\mathscr{E}}(\omega)|G(\omega)|^2 \, d\omega \tag{21}$$

where $\gamma_{\mathscr{E}}(\omega)$ is the incident field spectrum before filtering. In the limit where $t_0 = -\infty$, we use equations (12) and (13) to obtain from equation (17)

$$w_b^{(1)}(0) = \int_{-\infty}^{+\infty} S(\theta)\Gamma_{\mathscr{E}}(\theta) \, d\theta + \text{CC} = \int_{-\infty}^{+\infty} (S(\omega) + S^*(\omega))\gamma_{\mathscr{E}}(\omega) \, d\omega,$$

where CC stands for complex conjugate.

In conclusion, if we choose the atomic response $S(\omega)$ satisfying equation (20), the counting rate $w^{(1)}(t)$ tends to the same value for $t_0 \to -\infty$, where the EM field is filtered by the detector or by placing a filter in front of it.

3. *N*-fold coincidences

With the previous hypothesis (G_3) as in § 2, the probability that n photons are absorbed from the field by n atoms of the detector ionized from their ground state ($|\{g\}\rangle \to |\{a_j\}\rangle$), is given by the n-fold term of perturbation theory. We can thus show that the probability transition amplitude from an initial state $|i\rangle$ of the field to a final state $|f\rangle$ having n photons less, is still the matrix element of equation (5.7) (Glauber 1965)

$$\langle f, \{a_j\}|U^{(n)}(t, t_0)|i, \{g\}\rangle = \int_{t_0}^{t} \cdots \int_{t_0}^{t} \exp\left(\sum_j i\omega_{a_j g} t_j\right) \langle f|E^+(t_n) \ldots E^+(t_1)|i\rangle \prod_j M_{a_j g} \, dt_j. \tag{23}$$

The absorption probability of n photons, $P^{(n)}\{t_j\}$ between (t_0, t_j) $(j = 1, \ldots, n)$, is obtained in the same manner as $P^{(1)}(t)$ (cf § 2), we have to start from the square modulus of equation (23), sum it over all the final states of the field, take the mean value over the initial states $|i\rangle$ of the field and weight the new expression by the functions $R(a_j)$ which depend on the geometrical characteristics of the detector. Then we obtain

$$P^{(n)}\{t_j\} = \int_{t_0}^{t_1} \cdots \int_{t_0}^{t_n} \left(\prod_{j=1}^{n} S(t_j'' - t_j')\right) G^{(n)}(\{t_j'\}; \{t_j''\}) \, dt_j' \, dt_j'' \tag{24}$$

where

$$G^{(n)}(\{t_j'\}, \{t_j''\}) = \text{Tr}(\rho E^-(t_1') \ldots E^-(t_n') E^+(t_n'') \ldots E^+(t_1'')), \tag{25}$$

and $S(\theta)$ is still defined by equations (11), (12).

(*a*) For an ideal photodetector which receives a quasimonochromatic signal over a broad band level, $\Delta\omega \gg \delta\omega$, we put $S(\theta) = \delta(\theta)$ in equation (24) which becomes equation (5.12) of Glauber (1965),

$$P^{(n)}\{t_j\} = \int_{t_0}^{t_1} \cdots \int_{t_0}^{t_n} G^{(n)}(\{\theta_i\}, \{\theta_i\}) \prod_i \mathrm{d}\theta_i. \tag{26}$$

The n-fold coincidence probability at time $\{t_i\}$ is generally related to $P^{(n)}$ by the relation

$$w^{(n)}\{t_i\} = \frac{\partial^n P^{(n)}\{t_i\}}{\partial t_1 \ldots \partial t_n}. \tag{27}$$

For an ideal photodetector, equation (26) gives

$$w^{(n)}\{t_i\} = G^{(n)}(\{t_i\}, \{t_i\}).$$

(*b*) For an arbitrary detector for which $\Delta\omega$ is not much greater than $\delta\omega$, the n-fold coincidence rate is obtained from equations (24), (27)

$$w^{(n)}\{t_i\} = \sum_{\substack{\text{couples } (t_i, t_i') \\ \text{and } (t_i', t_i)}} \int_{t_0}^{t_1} \cdots \int_{t_0}^{t_n} \prod_i \mathrm{d}t_i' S(t_i^\alpha - t_i^\beta) G^{(n)}(\{t_i^\alpha\}, \{t_i^\beta\}). \tag{29}$$

In the integration over the variable t_i' ($i = 1, \ldots, n$), we introduced the indexes α and β to signify that the couple (t_i^α, t_i^β) is either (t_i, t_i') or (t_i', t_i). The second member of equation (29) is thus a sum of 2^n terms corresponding to all possibilities of the ensemble $\{t_1 \ldots t_n, t_n' \ldots t_i'\}$ where t_i and t_i' can be exchanged for every $i = 1 \ldots n$.

We notice that the n-fold coincidence rate is a real function, for

$$G^{(n)}\{t_i^\beta, t_i^\alpha\} = (G^{(n)}(t_i^\alpha, t_i^\beta))^*,$$

and $S(t)$ satisfies equation (13); $w^{(n)}\{t_i\}$ can thus be expressed as a sum of 2^{n-1} terms associated with their complex conjugates. For example, for $n = 2$,

$$\begin{aligned} w^{(2)}(t_1, t_2) = &\int_{t_0}^{t_1} \mathrm{d}t_1' \int_{t_0}^{t_2} \mathrm{d}t_2' S(t_1 - t_1') S(t_2 - t_2') G^{(2)}(t_1, t_2; t_2', t_1') + \text{CC} \\ &+ \int_{t_0}^{t_1} \mathrm{d}t_1' \int_{t_0}^{t_2} \mathrm{d}t_2' S(t_1 - t_1')\, S^*(t_2 - t_2') G^{(2)}(t_1, t_2'; t_2, t_1') + \text{CC}. \end{aligned} \tag{30}$$

Equations (28) and (29) are identical when we set $S(t) = \delta(t)$ in equation (29), since $\int_0^\infty f(t)\,\delta(t)\,\mathrm{d}t = \frac{1}{2} f(0)$. Each of the 2^n terms of equation (29) is equal to $(1/2^n) G^{(n)}\{t_i, t_i\}$.

Let us return now to the two experiments schematized in figure 2 and compare the results we should obtain.

If $\mathscr{E}(t)$ is the analytic signal (AS) of the incident EM field, $R(t)$ the response of the optical filter introduced in figure 2(*a*) before detection, with gain $G(\omega)$, the n-fold coincidence rate at time $\{t_i\}$ in case (*a*) is given by equation (28),

$$\begin{aligned} w_a^{(n)}\{t_i\} &= \langle \Pi_i |(\mathscr{E} * R)(t_i)|^2 \rangle \\ &= \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} \prod_{i=1}^{n} R(t_i - \theta_i) R^*(t_i - \theta_i')\, \mathrm{d}\theta_i\, \mathrm{d}\theta_i' \langle \mathscr{E}(\theta_1) \ldots \mathscr{E}(\theta_n) \mathscr{E}^*(\theta_n') \ldots \mathscr{E}(\theta_1') \rangle. \end{aligned} \tag{31}$$

In the case of experiment (*b*) where the EM field $\mathscr{E}(t)$ is not filtered before detection, the n-fold coincidence rate is given by equation (29) and

$$w_b^{(n)}\{t_i\} = \sum_{\substack{\text{couples}(t_i,t_i') \\ \text{and } (t_i',t_i)}} \int_{t_0}^{t_1} \dots \int_{t_0}^{t_n} \left(\prod_{i=1}^{n} \mathrm{d}t_i' \, S(t_i^\alpha - t_i^\beta) \right) \langle \mathscr{E}(t_1^\alpha) \dots \mathscr{E}(t_n^\alpha) \mathscr{E}^*(t_n^\beta) \dots \mathscr{E}^*(t_1^\beta) \rangle. \tag{32}$$

To compare equations (31) and (32), let us introduce the two following functions:

$$\begin{aligned} \mathscr{E}_{\mathrm{F}}(t) &= \int_{-\infty}^{t} \mathscr{E}(\theta) R(t-\theta) \, \mathrm{d}\theta \\ \mathscr{E}_{\mathrm{D}}(t) &= \int_{-\infty}^{t} \mathscr{E}(\theta) \, S^*(t-\theta) \, \mathrm{d}\theta, \end{aligned} \tag{33}$$

where $S^*(t) = S(-t)$. These random functions result from linear filtering of the AS of the broad-band incident EM field through two linear filters. The filter F whose response function is $R(t)$, and gain $G(\omega)$ gives $\mathscr{E}_{\mathrm{F}}(t)$; the filter D whose response is $S_1(t) = Y(t)S^*(t)$ gives $\mathscr{E}_{\mathrm{D}}(t)$. (We call $Y(t)$ the Heaviside step function introduced here to impose $t > 0$. Its gain should be $G_1(\omega) = S(-\omega) * \mathrm{vp}(1/\omega)$ where $\mathrm{vp}(1/x)$ is the distribution which corresponds to the Fourier transform of $Y(t)$.) Moreover, let us introduce the inter-correlation functions

$$G^{(n)}_{\mathscr{E}_{d_1}, \mathscr{E}^*_{\delta_1}}\{t_i, t_i\} = \langle \mathscr{E}_{d_1}(t_1) \dots \mathscr{E}_{d_n}(t_n) \mathscr{E}^*_{\delta_n}(t_n) \dots \mathscr{E}^*_{\delta_1}(t_1) \rangle \tag{34}$$

where each couple $(\mathscr{E}_{d_i}(t_i), \mathscr{E}^*_{\delta_i}(t_i))$ is either $(\mathscr{E}_{\mathrm{D}}(t_i), \mathscr{E}^*(t_i))$ or $(\mathscr{E}(t_i), \mathscr{E}^*_{\mathrm{D}}(t_i))$, the indexes d_i and δ_i indicate whether we consider the incident random function $\mathscr{E}(t_i)$ or the filtered one $\mathscr{E}_{\mathrm{D}}(t_i)$.

With these notations, we can write the n-fold coincidence probability, for experiments schematized in figures 2(*a*) and 2(*b*), if we suppose that the detector is illuminated since $t_0 = -\infty$,

$$\begin{aligned} w_a^{(n)}\{t_i\} &= G^{(n)}_{\mathscr{E}_{\mathrm{F}}, \mathscr{E}^*_{\mathrm{F}}}(\{t_i\}, \{t_i\}) \\ w_b^{(n)}\{t_i\} &= \sum_{2^n \text{ terms}} G^{(n)}_{\mathscr{E}_{d_i}, \mathscr{E}^*_{\delta_i}}(\{t_i\}, \{t_i\}) \\ (\mathscr{E}_{d_i}, \mathscr{E}^*_{\delta_i}) &= (\mathscr{E}, \mathscr{E}^*_{\mathrm{D}}) \qquad \text{or} \qquad (\mathscr{E}_{\mathrm{D}}, \mathscr{E}^*). \end{aligned} \tag{35}$$

The last results (equation (35)) coincide for $n = 1$, when we detect one photon, but do not coincide in the general case.

For the ideal detector which receives a quasimonochromatic field $\mathscr{E}_{\mathrm{F}}(t)$, the n-fold coincidence rate is a $2n$-moment of the narrow-band random function $\mathscr{E}_{\mathrm{F}}(t)$, but in case of experiment (*b*) where the narrow-band detector receives broad-band light, the n-fold coincidence rate is a sum of $2n$-moments where n narrow-band random functions $\mathscr{E}_{\mathrm{D}}(t)$ are associated with n broad-band random functions $\mathscr{E}(t)$.

4. Photocounting distribution

From the n-fold coincidence probability distributions Glauber (1965) calculated the probability distribution $p(m, T)$ that m photoelectrons are ejected by the photodetector during a time interval $(t_0, t_0 + T)$. He showed that, with the hypotheses G_1, G_2, G_3 stated in § 2 (cf equations (2), (3)), the emitted photoelectrons are distributed in time as a compound Poisson point process.

If we maintain the condition G_3 (equation (3)), ie we do not consider multiphoton processes, but without any restriction on the spectral width of the incident beam, we can show that Glauber's result is still valid and that the photocounting distribution is given by (cf equation (17.35), Glauber 1965),

$$p_{t_0}(m, T) = \int P\{\alpha_k\} \exp(-\Omega_T\{\alpha_k\}) \frac{\Omega_T^m\{\alpha_k\}}{m!} \prod_k d^2\alpha_k, \tag{36}$$

with

$$\Omega_T\{\alpha_k\} = \int_{t_0}^{t_0+T} \mathscr{E}^*(t', \{\alpha_k\}) \mathscr{E}(t'', \{\alpha_k\}) S(t''-t')\, dt'\, dt''. \tag{37}$$

In this equation, $\mathscr{E}(t, \{\alpha_k\})$ has the same meaning as in equation (19) of § 2; and we have assumed the detector is point-like to simplify the notation and suppress the spatial integration over its volume.

Equation (37) shows that the point process associated with the photoelectrons is a compound Poisson process whose density is

$$\rho_{t_0}(t) = \int_{t_0}^{t} \mathscr{E}(t)\mathscr{E}^*(\theta) S^*(t-\theta)\, d\theta + \text{cc}. \tag{38}$$

This result corresponds to an experiment where the detector is only irradiated during the time interval (t_0, t_0+T) in these conditions $p(m, t)$ represents the probability distribution that during this time interval, T, the detector absorbs m photons and emits m photoelectrons by ionization.

In the usual counting experiment, the detector receives the incident light during a long time interval $(-\infty, +\infty)$ and we study the point process associated with the emitted photoelectrons. For example, we determine the distribution $p(m, t)$ that m photoelectrons are emitted between (t_0, t_0+T) for continuous radiation. In this case, we have to set $t_0 = -\infty$ in equation (29) so that the n-fold coincidence rate is given by equation (35). The probability distribution that n photoelectrons are emitted during the intervals $\{t_0, t_0+T_i\}$ is then obtained from equation (29) with $t_0 = -\infty$,

$$P^{(n)}\{T_i\} = \prod_i \int_{t_0}^{t_0+T_i} dt_i \int_{-\infty}^{t_i} dt_i' \sum_{\substack{(t_i^\alpha, t_i^\beta)=(t_i, t_i') \\ \text{and } (t_i', t_i)}} S(t_i^\alpha - t_i^\beta) G^{(n)}(t_i^\alpha, t_i^\beta). \tag{39}$$

We can notice that equations (26) and (39) are identical when $S(t''-t') = \delta(t''-t')$, ie for the ideal photodetectors. This property is easily understood because an ideal photodetector acts instantaneously so that it gives the same results whether it is receiving radiation from time t_0 or $t_0 = -\infty$.

In the general case of a non-instantaneous detector, we have to take care of the time at which the detector is put in the light beam. Now we only consider equation (39) which corresponds to the usual experiments, in this case the counting distribution established by Glauber (equation (36)) is still valid but we have

$$\Omega_T\{\alpha_k\} = \int_{t_0}^{t_0+T} \rho(t)\, dt, \tag{40}$$

with

$$\rho(t) = \int_{-\infty}^{t} \mathscr{E}(t) S(t-\theta) \mathscr{E}^*(\theta)\, d\theta + \text{cc}. \tag{41}$$

In conclusion, a photodetector with an arbitrary atomic impulse irradiated by an arbitrary EM field with condition G_3, emits photoelectrons according to a compound Poisson process whose density is

$$\rho(t) = \mathscr{E}(t)\mathscr{E}_D^*(t) + \mathscr{E}_D(t)\mathscr{E}^*(t), \tag{42}$$

where $\mathscr{E}_D(t)$ is defined by equation (33).

For an ideal photodetector equation (42) reduces to equation (1).

This density $\rho(t)$ (equation (42)), which is the signal obtained from the photodetector can be interpreted as the detected light intensity.

5. Properties of the signal $\rho(t)$

We study here some properties of the density $\rho(t)$ of the compound Poisson process emitted by a detector which obeys hypothesis G_3. We use the classical formalism with random functions, but our results are quite true in quantum mechanics.

5.1. Coherence time

Equation (42) shows firstly that the signal $\rho(t)$ is the real part of a product of two functions: $\mathscr{E}(t)$ whose coherence time is $\tau_c(\tau_c = 1/\delta\omega$, cf figure 1), and $\mathscr{E} * S^* = \mathscr{E}_D(t)$ whose coherence time is $T_D \sim \sup(\tau_c, T_a)$ where $T_a = 1/\Delta\omega$ (cf figure 1). For example, in the case of figure 2(*b*), where a narrow-band detector receives a broad-band light, the signal obtained is thus the product of a quickly fluctuating function ($\sim \tau_c$) and a slowly fluctuating function ($\sim T_a$), the signal will thus have a coherence time of about τ_c, and the detector will be able to follow the light fluctuations, however rapid they are. In every case, the coherence time of $\rho(t)$ is the same as the coherence time τ_c of $\mathscr{E}(t)$.

We must nevertheless notice that in this case $S(t)$ is constant during a time interval of about τ_c, so that $\mathscr{E}_D(t)$ and the signal both vanish. This property limits the scope of our conclusion. We shall now give an order of magnitude estimate of the signal.

From equations (38) and (13) we have

$$\langle \rho(t) \rangle = \int_{-\infty}^{+\infty} S_1(u)\Gamma_{\mathscr{E}}(u)\,du + \text{CC}. \tag{43}$$

where $S_1(t)$ is the causal function $Y(t)S^*(t)$.

In the case of figure 2(*b*), the response function $S_1(u)$ is constant over a time interval of order τ_c, therefore we have approximately

$$\langle \rho(t) \rangle \sim S(0)\tau_c\Gamma_{\mathscr{E}}(0). \tag{44}$$

In equation (44) we used the relation $\int_{-\infty}^{\infty} \Gamma(u)\,du \sim \tau_c\Gamma(0)$ which is only rigorous for a rectangular correlation function of width τ_c. Let us call $s = S(\omega = 0)$ and $T_a = 1/\Delta\omega$. Thus we have

$$S(0) \sim \frac{s}{T_a}, \tag{45}$$

and

$$\langle \rho(t) \rangle \sim s\frac{\delta\omega}{\Delta\omega}\Gamma_{\mathscr{E}}(0) = s\frac{\tau_c}{T_a}\Gamma_{\mathscr{E}}(0). \tag{46}$$

This equation shows clearly that the signal is very weak in the case given by figure 2(*b*), a narrow-band detector which receives a broad-band light emits a signal $\delta\omega/\Delta\omega$ times smaller than the intensity emitted by an ideal detector receiving the same light. This result is not surprising, for in case shown in figure 2(*b*), the detector does not observe the greatest part of the energy of the incident light.

5.2. *Bunching effect*

Let us derive the bunching effect (Glauber 1965) of photoelectrons emitted by our non-ideal photodetector; it is commonly characterized by the normalized intensity correlation function

$$h(\tau) = \frac{\langle \rho(t)\rho(t-\tau)\rangle}{\langle \rho(t)\rangle^2}. \tag{47}$$

In our case the signal correlation function can be obtained from equation (41):

$$\begin{aligned}\Gamma_\rho(\tau) &= \langle \rho(t)\rho(t-\tau)\rangle \\ &= \iint_{-\infty}^{+\infty} \mathrm{d}\theta_1\,\mathrm{d}\theta_2\, S_1(t-\theta_1)S_1(t-\tau-\theta_2)\langle \mathscr{E}(t)\mathscr{E}(t-\tau)\mathscr{E}^*(\theta_1)\mathscr{E}^*(\theta_2)\rangle + \text{CC} \\ &\quad + \iint_{-\infty}^{+\infty} \mathrm{d}\theta_1\,\mathrm{d}\theta_2\, S_1(t-\theta_1)S_1^*(t-\tau-\theta_2)\langle \mathscr{E}(t)\mathscr{E}(\theta_2)\mathscr{E}^*(\theta_1)\mathscr{E}^*(t-\tau)\rangle + \text{CC}.\end{aligned} \tag{48}$$

In order to calculate $\Gamma_\rho(\tau)$, knowledge of the spectrum of the incident field is not sufficient and we have to know more about the statistical properties of $\mathscr{E}(t)$ since $\Gamma_\rho(\tau)$ contains fourth-order moments of the field.

Let us estimate the order of magnitude of $\Gamma_\rho(\tau)$ in the case of a chaotic field for which every $2n$-moment of $\mathscr{E}(t)$ can be written as a sum of n products of second-order moments $\Gamma_{\mathscr{E}}(\tau)$,

$$\begin{aligned}\Gamma_\rho^{(\mathrm{ch})}(\tau) &= \left(\int_0^\infty S(u)\Gamma_{\mathscr{E}}(u)\,\mathrm{d}u\right)^2 + \text{CC} + 2\int_0^\infty S(u)\Gamma_{\mathscr{E}}(u)\,\mathrm{d}u \int_0^\infty S^*(u)\Gamma_{\mathscr{E}}^*(u)\,\mathrm{d}u \\ &\quad + \int_0^\infty S(u)\Gamma_{\mathscr{E}}(u-\tau)\,\mathrm{d}u \int_0^\infty S(u)\Gamma_{\mathscr{E}}(u+\tau)\,\mathrm{d}u + \text{CC} \\ &\quad + \Gamma_{\mathscr{E}}(\tau) \iint_\infty^\infty S(t-\theta_1)S^*(t-\tau-\theta_2)\Gamma_{\mathscr{E}}(\theta_2-\theta_1)\,\mathrm{d}\theta_1\,\mathrm{d}\theta_2 + \text{CC}.\end{aligned} \tag{49}$$

In equation (49), the first three terms are $(\langle \rho(t)\rangle\langle \rho(t-\tau)\rangle)$ (see equation (43)), the fourth and fifth terms are of order $|S(\tau)|^2(\frac{1}{2}\tau_c|\Gamma_{\mathscr{E}}(0)|)^2$, and the two last are of order

$$\Gamma_{\mathscr{E}}(\tau) \iint S(u)S^*(u')\Gamma_{\mathscr{E}}(u-u')\,\mathrm{d}u\,\mathrm{d}u' \sim \Gamma_{\mathscr{E}}(\tau)\Gamma_{\mathscr{E}}(0)\tau_c \int |S^2(u)|\,\mathrm{d}u.$$

We can use Parseval's theorem and equation (45) to set

$$\int |S(u)|^2\,\mathrm{d}u \sim \int S(\omega)^2\,\mathrm{d}\omega \sim s^2\frac{1}{T_a}.$$

Then we have (cf figure 3), $h^{(ch)}(\infty) = 1$ and

$$h^{(ch)}(0) = \frac{\Gamma_\rho(0)}{\langle\rho\rangle^2} \sim \frac{3}{2} + \frac{T_a}{\tau_c} \sim \frac{\delta\omega}{\Delta\omega}. \tag{50}$$

This relation shows that in case of the figure 2(*b*) experiment, the photoelectron bunching effect is much greater than it would be in case of figure 2(*a*) for the same chaotic incident field. The normalized intensity correlation function has a maximum $h(0)$ which is of order of the spectral width ratio between the incident field $\delta\omega$ and the detector $\Delta\omega$, which is much greater than unity in case of figure 2(*b*). While in case of figure 2(*a*), $h(0) = 2$ for the same chaotic incident field.

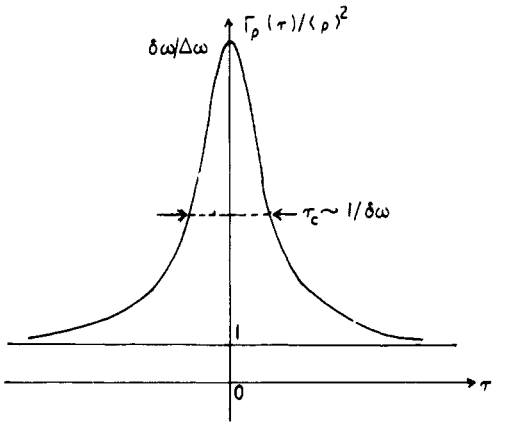

Figure 3. Normalized correlation function $h(\tau)$ of the signal emitted by a non-ideal photodetector. $h(\infty) = 1$, $h(0) = \delta\omega/\Delta\omega$ for a chaotic field. The width of $h(\tau)$ is of order $\tau_c = 1/\delta\omega$.

We must emphasize that the width of $\Gamma_\rho(\tau)$ or $h(\tau)$ is equal to τ_c. Therefore in order to measure $h(\tau)$ with the actual electronic performance we should use an incident field $\mathscr{E}(t)$ whose coherence time is not smaller than 10^{-11} s ($\tau_c \geqslant 10^{-11}$ s, $\delta\omega \leqslant 6 \times 10^{11}$ Hz), and therefore a detector with spectral width $\Delta\omega_a < 10^{11}$ Hz. Let us discuss this point. There are two sorts of photodetectors, those acting by photoemission (for example photomultipliers) and those acting by photoconductivity (for example photodiodes) The spectral response is much narrower in the case of photoconductivity, the semiconductor B-Ge is sensitive from 20 μm to 50 μm, its spectral width is about 10^{13} Hz (Lax and Mavroides 1967).

The order of magnitude of $\Delta\omega_a$ for such photodetectors is not very different from the actual electronic performance. It is almost reasonable to hope we shall be able to perform a photodetection under the conditions given in figure 1(*b*) and verify our predictions.

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